Basic laws of turbulent mixing in the surface layer of the atmosphere

A.S. Monin and A.M. Obukhov*

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The legibility of the available document is very poor in places and some of its scientific terminology is a little awkward. To solve the first problem, and make some improvements on the second, reference was made to a second translation obtained from Frank Bradley of CSIRO. The only indication of the source of this second translation is the inscription 'L.S.G 13.3.1957' at its foot. This second translation was also used to cross-check some minor edits done during retyping into LaTeX. The references and Figures are from the L.S.G. document (the former because Miller leaves these in the original Russian, which is too hard to read given the poor reproduction, and the latter because they are clearer in the L.S.G. translation). Even so, this document remains basically a transfer of the Miller translation into LaTeX. I thank John Wilson for his careful proof reading.*

Keith McNaughton, 12/11/2008.

*The translator may be LSG Kovasznay of Johns Hopkins University, who was active in turbulence research at that time and translated at least one other paper on turbulence from Russian to English. Kovasznay met A.M. Obukhov during the latter's visit to the University in about 1957, so the translation may have been done either as preparation for, or in the aftermath of, their meeting.

Abstract

The article contains an analysis of the processes of mixing in a turbulent atmosphere, based on systematic application of the methods of the theory of similitude. Empirical data on the distribution of wind velocity under various conditions of temperature stratification are generalized and a method is proposed for computing the austausch characteristics on the basis of measuring wind velocity and temperature gradient.

Introduction

The questions of the physics of the surface layer have occupied a considerable place in meteorological investigations during the past 10-15 years. The laws of the processes in the surface layer are of interest not only to agrometeorology, which studies the effect of a "meteorological medium" on the growth of vegetation, but they also have a general geophysical significance, since the dynamic interaction of the atmosphere and the substrate, the "feeding" of the atmosphere by moisture and heat, is realized through the surface layer.

A large amount of research in the field of surface-layer physics has been done at the Main Geophysical Observatory; the works of S.A. Sapozhnikova [1], D.L. Laikhtman and A.F. Chudnovskii [2], M.I. Budyko [3] and M.P. Timofeev [4] are well known to Soviet meteorologists.

This research has provided valuable observational data on the distribution of wind, temperature and humidity in the surface layer, and a number of specific propositions have been drawn up on the methodology for computing turbulent austausch characteristics (Budyko, Laikhtman).

In this regard there are still a number of debatable questions in the theory of surface-layer mixing. The simplest system of the "logarithmic boundary layer", borrowed from technical aerodynamics, describes quite well the phenomena in a neutrally-stratified atmosphere, and is supported by much empirical data. However, this system is insufficient for describing processes in the real atmosphere where the temperature inhomogeneity is an essential factor influencing the development of turbulence. This latter fact (the temperature inhomogeneity) determines the specific nature of the problem of atmospheric turbulence as applied to surface-layer physics.

The works of Laikhtman [5] and Budyko [3], as well as those of a number of foreign researchers (Sverdrup, Rossby, Montgomery; see, e.g. [6]) have been devoted to computing the influence of temperature stratification on turbulent exchange. The individual results of these works contradict one another; in many respects the physical sense of the initial hypotheses is not clear. Thus, e.g. Budyko proposes that the atmospheric stratification be considered within the framework of the simplest system of the logarithmic boundary layer, formally replacing the Karman "universal constant" by a variable parameter, a function of stratification. In Budyko's system the basic characteristic of the substrate, roughness, is also a function of meteorological conditions. The purely formal nature of those relations is one of the shortcomings of Budyko's system. It should also be noted that the observed profiles of wind distribution with height regularly deviate from the logarithmic law during stratification conditions which differ from neutral equilibrium.

Laikhtman proposes a more complete method of approximating wind and temperature profiles (an exponontial law with a variable exponent), which makes it possible to discern the nature of deviations from the logarithmic law under various conditions of atmospheric stratification. However, Laikhtman's system contains too many free parameters which have to be determined in each individual case. This creates difficulties familiar in determining these parameters from empirical data and decreases the computational accuracy.

These critical remarks by no means are meant to detract from the value of the results obtained by Budyko and Laikhtman when solving individual problems; however, they indicate the necessity of devoloping the theory further and making the initial physical hypotheses more exact.

When analyzing the highly complex phenomena of surface-layer turbulence, where the temperature factors play an essential role, it is expedient to use the methods of the theory of similitude which are widely used in applied aerodynamics and thermal physics, and are the generally-accepted method of investigation in this area.

In 1943, A.M. Obukhov attempted to apply methods of the theory of similitude to problems of surface-layer physics [7]. The results obtained in this work were subsequently developed by A.S. Monin [8]. The theory developed in [7] and [8] evidently gives a satisfactory qualitative description of the processes.

Furthermore, the data used in [7] to determine the numerical parmeters in the proposed system were not sufficiently reliable (the critical Richardson number was mistakenly assumed to be 1/11, on the basis of Sverdrup's data), which made it impossible to make direct use of the formulas obtained in this work in actual computations.

The present work gives an analysis of the processes of turbulent mixing in the surface layer of the atmosphere on the basis of a systematic application of the methods of the theory of similitude, and the values of the numerical parameters are more exactly defined by using a sufficiently large amount of empirical data on gradient observations, obtained from the expeditions of the Main Geophysical Observatory and the Geophysical Institute of the Academy of Sciences of the USSR. On this basis, working formulas were obtained for computing the basic characteristics of the surface layer, viz., turbulent heat transfer, friction, the austausch coefficient, and moisture flux, from gradient measurement data. The computational method is illustrated by specific examples.

1 The logarithmic boundary layer

When analyzing the processes in the surface layer of the atmosphere on a theoretical basis, we will proceed from the generally accepted system of a flow over an infinite, rough surface whose horizontal properties are assumed to be quite uniform horizontally. The averaged characteristics of the flow in this system are a function only of the vertical coordinate z. The most important characteristics are the momentum, heat, and humidity fluxes.

The momentum flux can be treated as turbulent friction stress. Instead of turbulent friction

$$\tau = -\overline{\rho u' w'} \tag{1}$$

where u' and w' are the fluctuations of the horizontal and vertical wind velocity components, ρ is air density, and the bar indicates averaging, it is convenient to examine the dynamic velocity

$$v_* = \sqrt{\frac{\tau}{\rho}} \tag{2}$$

Within the confines of the surface layer, τ and the turbulent heat flux q can be considered to be practically independent of height z.

The condition that fluxes τ and q are constant (within the given tolerance) can serve to determine the actual concept of the surface layer. Let us attempt to give an approximate estimate of the height of the surface layer on the basis of possible changes in τ. We will proceed from the averaged equations of hydrodynamics in a Coriolis force field. The corresponding equation for the x-ooordinate (wind-velocity direction at the earth's surface) in a quasistationary case has the following forms

$$\frac{\partial \overline{u' w'}}{\partial z} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x} + \ell \, \overline{v} \tag{3}$$

where $\partial \overline{p} / \partial x$ is the pressure gradient, ℓ the Coriolis parameter and $\overline{v}$ the compontent of averaged wind velocity along the y-axis.

Let us integrate both sides of the equation with respect to height within the limits of a layer of thickness H and estimate the right-hand side:

$$\frac{\tau(0) - \tau(H)}{\rho} = \int_0^H \left| \frac{1}{\rho} \frac{\partial \overline{p}}{\partial x} - \ell \, \overline{v} \right| \mathrm{d}z < \int_0^H \frac{1}{\rho} \left| \frac{\partial \overline{p}}{\partial x} \right| \mathrm{d}z \tag{4}$$

The discarding of term $\ell\overline{v}$ leads to a strengthening of the inequality, since the Coriolis force partially compensates the effect of the pressure gradient. Introducing the dynamic velocity v_* and the geostrophic wind velocity $v_g = (1/\rho\ell)\,|\partial\overline{p}/\partial x|$, we can write the resultant inequality in the following form:

$$v_*^2(0) - v_*^2(H) < H\ell v_g \tag{5}$$

Let us define H such that the relative change of v_*^2 in a layer of thickness H does not exceed the tolerance a, i.e.,

$$\frac{v_*^2(0) - v_*^2(H)}{v_*^2(0)} \leq a \tag{6}$$

On the strength of inequality (5) it suffices that

$$H < \frac{av_*^2(0)}{\ell v_g} \tag{7}$$

in order that (6) be fulfilled. The ratio of friction velocity to geostrophic wind velocity can be estimated to be of the order of 0.05:

$$\frac{v_*}{v_g} \sim 0.05$$

from which it follows that

$$H \leq 2.5 \times 10^{-3} a \frac{v_g}{\ell}$$

When $v_g \sim 10$ m/sec and $\ell = 10^{-4}$ sec^{-1} we get

$$H \sim a \times 250\,\mathrm{m}$$

With a tolerance $a = 20\%$ we get the estimate of the height of the surface layer which we seek:

$$H = 50\,\mathrm{m}.$$

Within the limits of this layer, v_* can be considered practically constant and the effect of the Coriolis force (rotation of wind with height) can be neglected. The estimate obtained agrees quite well with observations.

Under conditions of neutral stratification the processes of turbulent mixing in the surface layer can be described by the logarithmic model of the boundary layer. The corresponding laws have been studied in detail in experimental aerodynamics, and are widely used in meteorology.

Let us recall the derivation of the logarithmic law of wind distribution on the basis of the hypothesis of similarity. Let us assume that for values of $z \gg h_1$, where h_1 is the height of the grass (the characteristic scale of the micro-inhomogeneities of the substrate), the statistical characteristics for relative movements in a stream are invariant with respect to the similarity transformation

$$x' = kx, \quad y' = ky, \quad z' = kz, \quad t' = kt.$$

In these transformations the half-space $z > 0$ transforms into itself, while the equations of motion remain unchanged. This condition is the theoretical basis for the assumed hypothesis of similitude. Let us also note that the natural scale of velocity $v_* = \sqrt{\tau/\rho}$ remains invariant with respect to the indicated transformations. Let us examine the stationary regime and establish a ratio of the difference of the averaged velocities at two levels z_2 and z_1 to the dynamic velocity v_*. The corresponding non-dimensional magnitude is a function of z_1 and z_2 and, on the strength of the assumption of self similitude of the flow, can be a function only of the ratio z_2/z_1:

$$\frac{\overline{v}(z_2) - \overline{v}(z_1)}{v_*} = f\,(z_2/z_1) \tag{8}$$

Let us determine the form of function $f(\zeta)$. Evidently for all three heights $z_3 > z_2 > z_1$

$$\overline{u}(z_3) - \overline{u}(z_1) = \overline{u}(z_3) - \overline{u}(z_2) + \overline{u}(z_2) - \overline{u}(z_1) \tag{9}$$

and along with this,

$$\frac{z_3}{z_1} = \frac{z_3}{z_2}\,\frac{z_2}{z_1} \tag{10}$$

From this it follows that function f satisfies the functional equation

$$\begin{gathered} f\,(\zeta_1\,\zeta_2) = f\,(\zeta_1) + f\,(\zeta_2) \\ (\zeta_1 = z_2/z_1, \quad \zeta_2 = z_3/z_2) \end{gathered} \tag{11}$$

The logarithmic function $f(\zeta) = C\,\mathrm{ln}\zeta$ is the only solution of this functional equation. Assuming $C = 1/\kappa$, we get

$$\frac{\overline{v}(z_2) - \overline{v}(z_1)}{v_*} = \frac{1}{\kappa}\ln\frac{z_2}{z_1} \tag{12}$$

where κ is the well-known Karman constant. According to empirical data, $\kappa \approx 0.4$. Equation (12) can be written in the usual differential form, examining the infinitely close values z_2 and z_1:

$$\frac{\mathrm{d}\overline{v}}{\mathrm{d}z} = \frac{v_*}{\kappa z} \tag{13}$$

Equations (12) and (13) do not contain charateristics of a particular substrate but can pertain to any substrate, if the condition $z_1, z_2 >> h_1$ is satisfied.[1] Then, too, formula (13) specifies only changes in mean wind velocity with height. The properties of the substrate must be considered in order to determine the absolute value of $\overline{v}(z)$.

Now let us assume that observations of wind velocity are conducted at a definite height H above some definite substrate. Let us assume that we can conduct independent measurements of the turbulent friction and, accordingly, in each individual case we can determine $\overline{v} = \sqrt{\tau/\rho}$. The value v_* can be determined, e.g. from thermo-anemometer meter observations of fluctuations u' and w', or directly on the basis of measurement of the drag intensity at the earth's surface. This latter method is used in practice when studying turbulent motion in tubes. Sheppard [9] attemptod to use the dynamometer method of measuring τ under atmospheric conditions.

A comparison of a number of observations of $\overline{v}(H)$ and τ allows us to determine the relationship between these magnitudes. Aerodynamic experiments teach us that with large Reynolds' nunbers and surface "roughness" the dependence of τ on $\overline{v}$ is of a quadratic nature, from which it follows that

$$v_* = \gamma(H)\, v(H) \tag{14}$$

where $\gamma(H)$ is a non-dimensional coefficient which is a function of the properties of the substrate. At a fixed height H the "drag coefficient" $\gamma(H)$ can serve as an objective characteristic of the properties of the substrate with respect to its dynamic influence on the flow. However, use of $\gamma(H)$ has the disadvantage that a specific observation height must be selected. The dependence of $\gamma(H)$ on the observation height H can be easily established by substituting $\overline{v}(H) = v_*/\gamma(H)$ in formula (12). For any two heights $H_1, H_2 >> h_1$ we will have

$$\frac{1}{\gamma(H_2)} - \frac{1}{\gamma(H_1)} = \frac{1}{\kappa}\ln\frac{H_2}{H_1} \tag{15}$$

From (15) it follows that, in particular, $\gamma(H)$ decreases with height. Taking the antilogarithms and combining the magnitides which contain H_1 and H_2 respectively, we get

$$H_1\, \mathrm{e}^{-\kappa/\gamma(H_1)} = H_2\, \mathrm{e}^{-\kappa/\gamma(H_2)} = h_o \tag{16}$$

i.e., a magnitude which is not a function of height. Thus the magnitude h_o, which has length, is determined only by the properties of the substrate; it is

[1] Determination of the values of height z in formula (13) involves a certain arbitrariness in the choice of the starting point for the computation (within the limits of the height of the grass h_1). However, when $z \gg h_1$, this indefiniteness is of no importance.

called "dynamic roughess". Let us express the drag coefficient $\gamma(z)$ in terms of h_o:

$$\gamma(z) = \frac{\kappa}{\ln \frac{z}{h_o}} \tag{17}$$

whence on the basis of (14) we get the desired wind velocity distribution:

$$v(z) = \frac{v_*}{\kappa} \ln \frac{z}{h_o} \tag{18}$$

The method given above for introducing the concept of roughness of the substrate has the advantage that it depends exclusively on the properties of the flow at rather great heights, where there are sufficient grounds for using the universal laws of developed turbulence. In most cases, however, we have no means for making direct measurements of τ (and, accordingly, $\gamma(H)$), and in this regard, when making practical determinations of the characteristics of dynamic roughness, we must use the properties of the wind profile which can be determined directly from observations. When dealing with a mature vegetation cover, additional difficulties arise in connection with choosing the reference height for specifying z. A number of authors (Paeschke [10] Konstantinov [11]) recommend the use of a certain arbitrary level z_1 for the start of height computations; this level lies between the soil and the top of the grass h_1. This level can be called the height of the displacement layer.

The concept of "displacement height" z_1 can be introduced into the general system as follows. Equation (13) describes an asymptotic relationship valid when $z >> h_1$, and in this region it is insensitive to slight changes in the reference point of z (within the limits of the grass heighth_1). Let us now examine the range of values of z which, although they exceed h_1, are nevertheless comparable with it, so the ratio h_1/z can be treated as a first-order value. To be specific, we will compute z from ground level. In this case, a numerical correction factor $f(h_1/z)$ should be introduced into formula (13); this describes the deviation from the automodular regime, connected with the direct effect of the grass:

$$\frac{\mathrm{d}\overline{v}}{\mathrm{d}z} = \frac{v_*}{\kappa z} f\left(\frac{h_1}{z}\right) \tag{19}$$

Evidently, when $z \to \infty$, formula (19) should convert into (13), from which it follows that $f(0) = 1$. Expanding function f in series, we get

$$\frac{\mathrm{d}\overline{v}}{\mathrm{d}z} = \frac{v_*}{\kappa z} \left[1 + \alpha(h_1/z) + \beta(h_1/z)^2 + \ \ldots \ \right] \tag{20}$$

Let us now introduce a new starting point for computations of z, assuming $z = z' + z_1$, where z' is comparable with h_1, and rewrite the equation with respect to the new variable:

$$\frac{\mathrm{d}\,\overline{v}}{\mathrm{d}z'} = \frac{v_*}{\kappa z'}\left[1 + \left(\alpha - \frac{z_1}{h_1}\right)(h_1/z') + \beta'(h_1/z')^2 + ...\right] \tag{21}$$

Let us select z_1 such that in equation (21) the first-order term reverts to zero. With a corresponding choice of z_1 with an accuracy up to the second-order terms, we get

$$\frac{\mathrm{d}\,\overline{v}}{\mathrm{d}z} = \frac{v_*}{\kappa(z - z_1)} \tag{22}$$

Thus, the height of the displacement layer can be defined as the height of some arbitrary level of computation, using which we get the best approximation of the wind profile by the logarithmic law in a layer situated above the grass layer. Let us note that the physical determination given above of dynamic roughness h_o is insensitive to a substitution of $z - z_1$ for z (Since $H >> h_1$); however, in the final formula for the wind velocity profile we should calculate the height from the level of the displacement layer, i.e., replace z by $z - z_1$:

$$\overline{v}(z) = \frac{v_*}{\kappa}\ln\frac{z - z_1}{h_o} \tag{23}$$

The characteristics of the substrate, z_1 and h_o can be determined empirically on the basis of measurements of wind profile in the layer above the grass level, under conditions close to equilibrium. To increase the computational accuracy we should use data averaged for a group of analogous cases.

Let us use, as an example, values of z_1 and h_o according to Paeschke's work [10] (table 1):

Table 1

Characteristics of the substrate

	z_1, cm	h_o, cm
Snow surface	3	0.5
Airport	10	2.5
Sugar beet plantation	45	6.6
Wheat field	130	5

Some data on the question of choosing the initial level z_1 can be found in an article by A. R. Konstantinov [11]. It is worth noting that the dynamic roughness of a wheat field is less than that of a sugar beet plantation, although the grass is three times higher in the first case. In the case of a low grass stand (steppe) the value of z_1 does not play an essential role, and when computing h_o and v_* from observations made at heights of more than 1 meter, we can consider formally that $z_1 = 0$, i.e., we can compute the height directly from the ground.

In further sections of this work, when considering the effects of stratification, we will assume that height is calculated from some arbitrary level ("the displacement layer"), not specifically mentioned, while the dynamic roughness length h_o will be computed by some given characteristic of the substrate which is independent of meteorological conditions.

2 Basic characteristics of the turbulent regime in a medium with non-uniform temperature

One of the most important practical characteristics of the turbulent regime in the surface layer of the atmosphere is the vertical turbulent heat flux:

$$q = c_p \rho \, \overline{w'T'} \tag{24}$$

where c_p is the specific heat of the air at constant pressure, ρ is density, w' and T' are, respectively, the fluctuations of the vertical wind velocity component and of temperature, caused by the turbulence 'elements' passing a given point, and the bar indicates averaging. The magnitude of q is the average amount of heat carried by turbulent fluctuations across a unit area per unit time. We have sufficient grounds for considering that for all intents and purposes the turbulent heat flux q in the surface layer under stationary conditions is not a function of height[2]. Instead of q we may use the "temperature flux"

$$\frac{q}{c_p \rho} = \overline{w'T'} \tag{25}$$

The magnitude of the turbulent heat flux q can be determined directly experimentally, on the basis of electronic measurements of the fluctuations of temperature T' and of the vertical wind velocity w'. Modern technology has shown that such measurements are possible, in principle [12, 13]. Nevertheless, in practice one must still use indirect methods to determine q, based on simpler gradient methods. To interpret these measurements correctly, one must investigate the connection between the characteristics of turbulence q and v_*

[2] Here we are digressing from an examination of radiational energy fluxes. Strictly speaking, the total flux $q + q_1$ is not a function of height; here q_1 is the radiation flux. Then, too, in the surface layer, changes in the radiation flux q_1 can hardly be considered essential. This question, however should be the subject of special investigations.

and the distribution of mean wind speed and temperature. When solving this problem we will follow the methods of the theory of similitude and attempt to establish a system with a minimum number of parameters which describe the turbulent regime in an inhomogeneous temperature medium.

The inhomogeneities of the temperature field, being of a systematic nature (change of mean temperature with height), exert a definite influence on the general turbulent regime (the effect of Archimedian forces). Provided that the temperature fluctuations are slight compared with the mean temperature of the layer T_o, the equations for the dynamics of an inhomogeneous temperature medium can be written in the following form:

$$\begin{aligned}
\frac{\mathrm{d}u}{\mathrm{d}t} &= -\frac{1}{\rho}\frac{\partial p_1}{\partial x} \\
\frac{\mathrm{d}v}{\mathrm{d}t} &= -\frac{1}{\rho}\frac{\partial p_1}{\partial y} \\
\frac{\mathrm{d}w}{\mathrm{d}t} &= -\frac{1}{\rho}\frac{\partial p_1}{\partial z} + \frac{g}{T_o}T_1 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &+ \frac{\partial w}{\partial z} = 0 \\
\frac{\mathrm{d}T_1}{\mathrm{d}t} &= 0
\end{aligned} \tag{26}$$

In this system p_1 and T_1 indicate deviations from the standard values.

The simplifications made when deriving the system of equations are: neglect of the Coriolis force and the radiation influx of heat, and also the linearization of the standard statistical distribution of pressure and temperature. This latter indicates that change in density due to pressure changes are neglected, and it assumes that the deviation of density and temperature from the standard values are proportional (L. D. Landau and E.M. Lifshitz [14, chapter 5]). These simplifications, used in the convection theory, allow us to describe the Archimedean force by the term $(g/T_o)\,T_1$. Thus, the equation contains a dimensional constant g/T_o, which we should consider in the future when establishing the criteria of similitude.

Let us note that we cannot linearize the equations of velocity variations, since in this case turbulence would be lost. In addition, in the equations, the terms containing viscosity and heat conductivity would be omitted[3]. It is natural to assume that changes in mean velocity and temperature with

[3]Under conditions of a developed turbulent regime, these terms must be considered only when investigating the very fine details of the microstructure of the wind and temperature fields. The vertical transport of momentum and heat is caused by the inhomogeneities of some "mean scale", for which the direct influence of viscosity and heat conductivity are rather slight.

height can be expressed by coordinate z, parameter g/T_o, and the "external parameters" v_* and q while the corresponding equations can be written in non-dimensional form, since they do not contain other dimensional constants. This proposition is the basic hypothesis of the theory of similitude, formulated in the first section of the present work, generalized for the case of a medium with a non-uniform temperature.

The similarity hypothesis which we have adopted agrees with equations (26) and is equivalant to the proposition that the sytem of equations (26) together with the conditions

$$\overline{w'\,T'} = \frac{q}{c_p\rho} = \text{const}$$
$$-\overline{\rho\,u'\,w'} = \tau = \text{const} \tag{27}$$

are an analogue of the boundary conditions and define the statistical characteristics of the turbulent regime unequivocally. Thus, the three parameters g/T_o, v_* and $q/c_p\rho$ can be considered the definitive characteristics of the turbulence of the surface layer (in the layer above the top of the grass). From these parameters we can establish unequivocally (with an accuracy of the numerical coefficients) the scale of length L and temperature T_*, which can be written in the following form:

$$L = -\frac{v_*^3}{\kappa \frac{g}{T_o} \frac{q}{c_p\rho}}, \qquad T_* = -\frac{1}{\kappa u_*} \frac{q}{c_p\rho} \tag{28}$$

It is natural to use dynamic velocity v_* as the characteristic velocity scale. The minus sign and the Karman constant κ are introduced for the sake of convenience. The signs of L and T_* are determined by the nature or the stratification. With stable stratification the turbulent heat flux is directed downward, $q < 0$, and correspondingly $L > 0$ and $T_* > 0$. With unstable stratification on the other hand, $q > 0$, $L < 0$ and $T_* < 0$. Thus, we must visualize two qualitatively different regimes, corresponding to the cases $q < 0$ and $q > 0$. These regimes should unite as conditions of neutral stratification ($q = 0$) are approached.

Let us examine the non-dimensional magnitudes $\left(\frac{\kappa z}{v_*}\frac{\mathrm{d}\overline{v}}{\mathrm{d}z}\right)$ and $\left(\frac{z}{T_*}\frac{\mathrm{d}\overline{T}}{\mathrm{d}z}\right)$ (from now on, the bar which indicates averaging will be omitted). These non-dimensional characteristics of the averaged field of velocities and temperatures should be definite functions of the "external parameters" and of coordinate z. The only non-dimensional combination which we can make from $q/c_p\rho$, v_*, g/T_o and z is z/L, from which it follows that

$$\frac{\kappa z}{v_*}\frac{\mathrm{d}v}{\mathrm{d}z} = \varphi_1\left(\frac{z}{L}\right) \tag{29}$$

$$\frac{z}{T_*}\frac{\mathrm{d}T}{\mathrm{d}z} = \varphi_2\left(\frac{z}{L}\right) \tag{30}$$

or

$$\frac{\mathrm{d}v}{\mathrm{d}z} = \frac{v_*}{\kappa z}\varphi_1\left(\frac{z}{L}\right) \tag{29$'$}$$

and

$$\frac{\mathrm{d}T}{\mathrm{d}z} = \frac{T_*}{z}\varphi_2\left(\frac{z}{L}\right) \tag{30$'$}$$

where T_* and L are determined by formula (28).

Let us introduce the concept of the austausch coefficient. Let us assume formally that

$$\begin{aligned} \tau &= \rho K \frac{\mathrm{d}v}{\mathrm{d}z} \\ q &= -c_p \rho K_T \frac{\mathrm{d}T}{\mathrm{d}z} \end{aligned} \tag{31}$$

and call the dynamic austausch coefficient and the coefficient of turbulent heat conductivity K and K_T respectively. Introducing the magnitudes $v_* = \sqrt{\tau/\rho}$ and $T_* = -\frac{1}{\kappa v_*}\frac{q}{c_p\rho}$ in place of τ and q, and using equations (29) aid (30), we get

$$K = \frac{\kappa\, v_*\, z}{\varphi_1\left(\frac{z}{L}\right)}, \qquad K_T = \frac{\kappa\, v_*\, z}{\varphi_2\left(\frac{z}{L}\right)} \tag{32}$$

Now let us examine the hypothesis, shared by a majority of meteorologists, that within the limits of meteorological observations we can consider that $K = K_T$[4] from which it follows that

$$\varphi_1\left(\frac{z}{L}\right) = \varphi_2\left(\frac{z}{L}\right) = \varphi\left(\frac{z}{L}\right) \tag{33}$$

The similitude of the temperature and wind profiles follows directly from the accepted hypothesis that $K = K_T$. Dividing (30) by (29) we get

$$\frac{\mathrm{d}T}{\mathrm{d}v} = -\frac{q}{c_p\tau} = \frac{\kappa T_*}{v_*} \tag{34}$$

[4]Generally speaking $K > K_T$ since the effect of pressure fluctuations, as well as mixing can be expressed in a momentum exchange. However, as of now we have no convincing evidence that this difference is essential. The theory developed in the present work can be generalized for the case $K/K_T = a \neq 1^*$ if we replace T by T/a in all instances.

and, accordingly, for any heights H_1 and H_2

$$T(H_2) - T(H_1) = \frac{\kappa T_*}{v_*} \left[\overline{v}(H_2) - \overline{v}(H_1)\right] \tag{35}$$

Thus, the ratio of the difference of mean temperature at two levels H_1 and H_2 to the difference in velocities at the same heights does not depend on the choice of heights H_1 and H_2, but is determined entirely by external conditions - the ratio of the turbulent heat flux q to the turbulent drag resistance τ.

Let us now show that the non-dimensional factor $\varphi(z/L)$, where $L = -v_*^3/\left(\kappa \frac{g}{T_o}\, \frac{q}{c_p \rho}\right)$, is directly connected with the Richardson number at any given level. Substituting the values $\mathrm{d}v/\mathrm{d}z$ and $\mathrm{d}T/\mathrm{d}z$, determined from formulas (29) and (30), in the expression for the Richardson number[5]

$$Ri = -\frac{g}{T_o} \frac{\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)}{\left(\frac{\mathrm{d}v}{\mathrm{d}z}\right)^2} \tag{36}$$

we get

$$Ri = -\frac{g\kappa^2}{T_o} \frac{T_* z}{v_*^2\, \varphi\left(\frac{z}{L}\right)} \tag{37}$$

or, using the definition of the scale of L by (28)

$$Ri = \frac{z}{L} \times \frac{1}{\varphi\left(\frac{z}{L}\right)} \tag{38}$$

from which it follows that the dependence of the Richardson number on height is defined by a single parameter—the scale L.

Comparing formula (32) for the austausch coefficient with the expression for the Richardson number, we get an important relationship between the austausch coefficient, the scale L and the Richardson number:

$$K = \kappa v_* L\, Ri \tag{39}$$

Let us explain the physical the meaning of the L scale. Under any conditions of stratification we have

$$\frac{\mathrm{d}v}{\mathrm{d}z} = \frac{v_*}{\kappa z}\, \varphi\left(\frac{z}{L}\right) \tag{40}$$

[5] It follows that T should indicate potential temperature, since T does not change with vertical shifts of the turbulent elements (the state of the latter can be considered adiabatic). In the surface layer the numerical values of potential and molecular temperature are very close. With the large temperature gradients usually observed in the surface layer, the difference between the gradients of potential and molecular temperature are inconsequential; however, in states close to isothermy, this difference is significant.

Let us fix the value z and decrease magnitude q indefinitely, approaching the conditions of neutral stratification, which correspond to infinite growth of the scale L (with respect to absolute magnitude). Obviously, within this range, we should obtain formula (22), from which it follows that

$$\varphi(0) = 1$$

Under given external conditions characterized by magnitudes v_* and q, and the corresponding magnitude of L, in the range of values of z which are quite small compared to L, $\varphi(z/L)$ will be quite close to unity. This indicates that austausch conditions with $z \ll L$ differ little from austausch conditions in a neutrally stratified atmosphere and, accordingly, turbulence is caused mainly by purely dynamic factors. Thus, the scale L, first introduced by Obukbov [7], is an important physical characteristic of the state of the surface layer and can be called the height of the sub-layer of dynamic turbulence. On the strength of the fact that $\varphi(0) = 1$ and formula (38), when $z \to 0$, we get

$$\frac{1}{L} = \left(\frac{\partial\,\mathrm{Ri}}{\partial\, z}\right)_{z=0} \tag{41}$$

This formula can serve as the basis for determining the scale L from empirical data (from the wind and temperature profiles).

The function $\varphi(z/L)$ should, in the general case, be determined from the aggregate of empirical data. It should be noted that the data available at present are insufficient to determine function φ reliably in a sufficiently wide range of changes of the argument z/L. However, a number of important problems can be solved for the case $z/L < 1$, where we can limit ourselves to the first terms of function φ expanded in series. This case requires special examination.

3 Determination of the turbulent characteristics from data on gradient measurements

In the case $|z/L| < 1$ we can limit ourselves to the first terms of the function $\varphi(z/L)$ expanded in a power series, and write

$$\varphi = 1 + \beta\,\frac{z}{L} \tag{42}$$

where β is some universal constant which can be determined on the basis of empirical data. From formulas (29), (30) and (42), by integrating with respect to z, we get

$$v(z) = \frac{v_*}{\kappa}\left[\ln\frac{z}{h_o} + \beta\,\frac{z}{L}\right]$$

$$T(z) - T(h_o) = T_*\left[\ln\frac{z}{h_o} + \beta\,\frac{z}{L}\right] \tag{43}$$

Here we have replaced the term $\beta[(z - h_o)/L]$ by $\beta(z/L)$, with the intention of using fornula (43) only when $z \gg h_o$.

Let us note that analogous formulas can be used to describe the profiles of the concentration of any passive substance in the surface layer of the atmosphere. For example, with a stationary turbulent regime with no phase transformation of the humidity in the atmosphere, the vertical moisture flux ("rate of evaporation") $E = \rho\,\overline{w'Q'}$ (Q is specific humidity) can be considered independent of height and, analogously to (30), we can set

$$\frac{\mathrm{d}Q}{\mathrm{d}z} = \frac{q_*}{z}\,\varphi\left(\frac{z}{L}\right), \qquad Q_* = -\frac{1}{\kappa v_*}\,\frac{E}{\rho}, \tag{44}$$

whence

$$Q(z) - Q(h_o) = Q\left[\ln\frac{z}{h_o} + \beta\,\frac{z}{L}\right] \tag{45}$$

Finally, the expression for the. austausch coefficient $K = \kappa v_* L\,\mathrm{Ri}$, following equation (38) and using the approximation (42), becomes

$$K(z) = \frac{\kappa\, v_*\, z}{1 + \beta\frac{z}{L}} \tag{46}$$

With neutral stratification ($|L| = \infty$) we get, from (43), the usual logarithmic formulas for wind and temperature distribution with height. Non-neutral stratification is described in (43) by the component $\beta(z/L)$ and leads to a systematic deviation from the logarithmic law. With unstable stratification ($L < 0$), intense turbulent mixing leads to equalization of wind velocity in different layers of the atmosphere, so that the wind velocity should increase with height more slowly than in the case of neutral stratification, i.e., $\beta(z/L)$ should be less than zero. Accordingly, $\beta > 0$.

Formulas (43) for $v(z)$ and $T(z)$ are in good qualitative (and, with correct selection of the parameters, also quantitative) agreement with the observed profiles of wind velocity and temperature in the surface layer. Actual measurements confirm the presence and nature of regular deviations of the logarithmic law in the wind and temperature distribution with height, indicated by formulas (43). This can be seen, e.g., from the data of Table 2, which shows wind

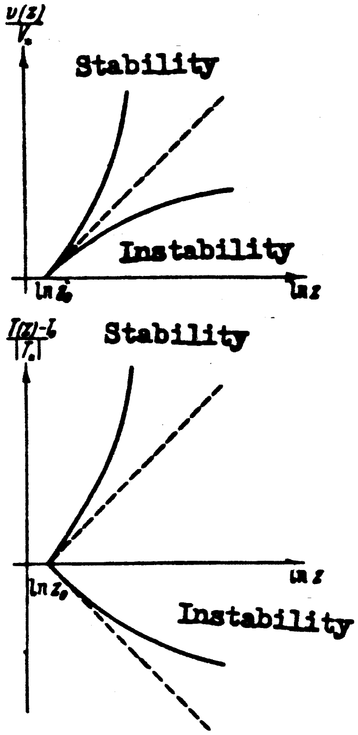

Figure 1: Nature of the wind and temperature profiles

profiles averaged by groups with an identical stability parameter $S = \frac{g}{T_o} \frac{\Delta T}{v^2}$ (taken from data of the Main Geophysical Observatory expedition of 1945 [15], 1947 [16] and 1950 [17] and the expedition of the Geophysical Institute of the Academy of Sciences of the USSR in 1951 [18]. The form of profiles $v(z)$ and $T(z)$, in agreement with formulas (43), is given in Figure 1. Figures 2 and 3 give the averaged profiles of wind velocity and temperature obtained by the 1951 expedition of the Geophysical Institute of the Academy of Sciences of the USSR.[6]

Halstead (19) proposed that the influence of stratification be computed by introducing correction factors into the logarithmic formulae, analogous to (43), but without analyzing the coefficients from the point of view of the theory of similitude.

Approximating the measured wind and temperature profiles by formulas

[6]The straight dashed lines in figures 1, 2 and 3 correspond to the logarithmic profile. The numbers -1, -2, . . . +3 correspond to the identification number of the group of profiles in Table 2.

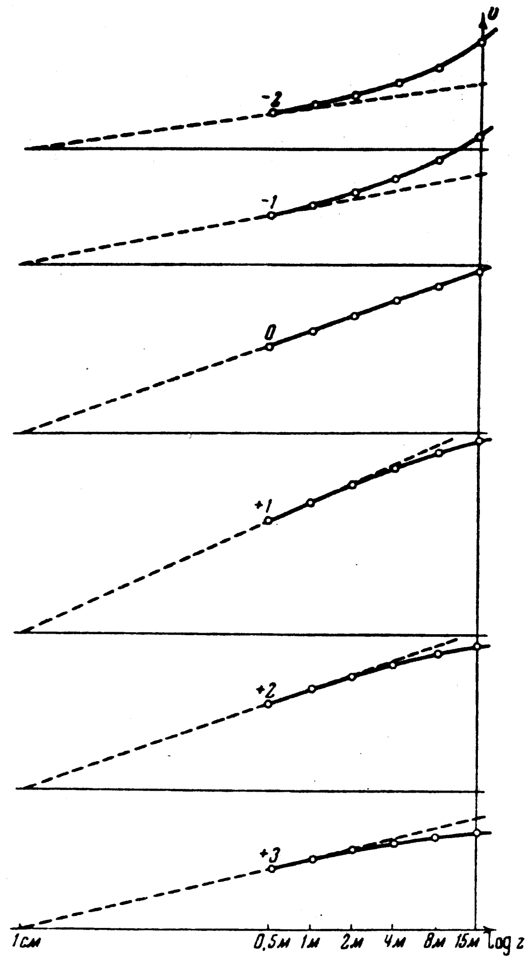

Figure 2: Averaged wind profiles from observational data.

(43), we can determine the turbulence characteristics from gradient measurement data. In practice, during such an approximation we must first determine the reference level for computing height z_1—the thickness of the displacement layer. The magnitude z_1 can be determined experimentally, so that on the graph with the logarithmic scale the wind profile, corresponding to cases of equilibrium stratification (i.e., actually, to cases of isothermy) would be depicted by straight lines with respect to height. Extrapolating the resulting rectilinear wind profile graph to zero velocity, we get the value of the roughness height h_o.

The magnitude h_o and the parameters v_*/κ and β/L which enter into formulas (43) can be most accurataly determined by using the least-squares

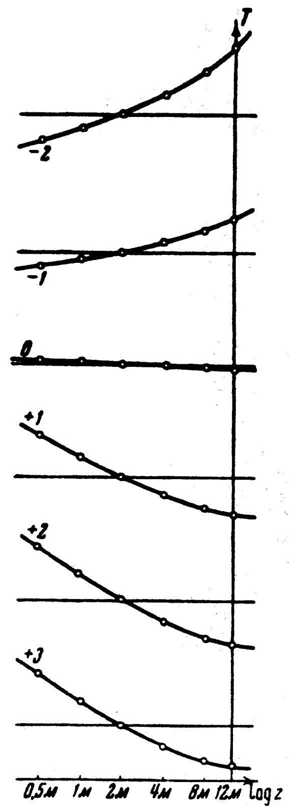

Figure 3: Averaged temperature profiles from observational data.

method to process the wind profile measured at the same station, generally speaking, under various conditions of stratificatlon. Thus, assuming

$$v_i(z) = A_i(\gamma + \log z) + C_i z$$

where i is the number of a profile, and selecting A_i, γ and C_i because of the requirement that the sum of the squares of the deviations be minimum,

$$\Delta^2 = \sum_{i,k} [A_i(\gamma + \log z_k) + C_i z_k - v_i(z_k)]^2 \tag{47}$$

we get for each profile

$$\frac{v_*}{\kappa} = \frac{A_i}{\ln 10}\,, \qquad \frac{\beta}{L} = \frac{C_i}{A_i}\ln 10$$

and we get a common roughness height $h_o = 10^\gamma$ for all profiles.

Having determined β/L for each profile by the indicated method, knowing h_o, and computing the value of the stability parameter $S = \frac{g}{T_o}\frac{T(z_1)-T(z_3)}{v^2(z_2)}$ (where e.g., $z_1 = 0.5$, m $z_2 = 1$ m, $z_3 = 2$ m), we can determine β using the formula

$$S = \frac{1}{\beta}\left[\frac{\beta}{L}\,\frac{\ln\, z_1/z_3}{\left(\ln\frac{z_2}{h_o}\right)^2}\;\frac{1+\frac{\beta}{L}\,\frac{z_1-z_3}{\ln\, z_1/z_3}}{\left(1+\frac{\beta}{L}\,\frac{z_2}{\ln\, z_2/h_o}\right)^2}\right] = \frac{1}{\beta}\;\Phi\left(\frac{\beta}{L}\right) \tag{48}$$

which follows from [43]. The number β can be determined as the regression coefficient of values of $\Phi(\beta/L)$, computed from the previously calculated β/L, for the computed values of S. The regression coefficient β, computed from the data of the four expeditions listed in Table 2, is 0.62; the accuracy in determining β in this case is probably not better than 10%. A determination of β from the data of just one Main Geophysical Observatory expedition [16] yielded a value of 0.57.

Using formulas (43) we can compute the drag velocity v_*, as well as the turbulence characteristic which has the most practical value, i.e. the heat flux q, using the results of wind velocity and temperature measurements at only two heights. For example, let $z_1 = H/2$, $z_2 = H$, $z_3 = 2H$ and let us assume the values $T_1 = T(z_1)$, $T_3 = T(z_3)$ and $v_2 = v(z_2)$ m/sec have been measured. Then from (43) we get

$$v_* = \frac{\kappa v_2}{\ln\frac{z_2}{h_o}\left(1+\frac{\beta}{\ln\frac{z_2}{h_o}}\,\frac{H}{L}\right)} = -\frac{0.19}{\log\frac{h_o}{H}}\;\frac{v_2}{\left(1-\frac{0.26}{\log\frac{h_o}{H}}\,\frac{H}{L}\right)} \qquad \text{m/sec}$$

$$q = -\frac{c_p\,\rho\,\kappa\,v_2\,(T_3-T_1)}{\ln\frac{z_3}{z_1}\left(1+\beta\,\frac{z_3-z_1}{H\ln\frac{z_3}{z_1}}\,\frac{H}{L}\right)} = -0.58\,\frac{v_2(T_3-T_1)}{1+0.65\,\frac{H}{L}} \qquad \text{Cal/cm}^2\text{/min} \tag{49}$$

Here we used the value $\kappa = 0.43$ for the von Karman constant. The magnitude of H/L is determined from the relationship (48), which assumes the form

$$\frac{L}{H} = \frac{0.26}{\log\frac{h_o}{H}} + \frac{1}{2B}\left[1+\sqrt{1+4B\left(0.65+\frac{0.26}{\log\frac{h_o}{H}}\right)}\right] \tag{50}$$

where $B = .107\,H\,(\log z_o/H)^2\,\left((T_3-T_1)/v_2^2\right)$ and H and v_2 are expressed in meters and m/sec, respectively.

The influence of stratification on the magnitudes v_* and q is expressed by the appearance of the components with H/L in the denominators of formulas

(49). As a rule, the correction for stratification appears to be slight ($H/L \sim 10^{-1}$), which is natural, since turbulence in the lower part of the surface layer is determined essentially by the dynamic factors.

Formulas (49) and (50) can be useful when mass-processing the gradient measurement data. They assume a relatively simple form in situations where h_o and H are fixed. For example, when $h_o = 1$ cm and $H = 1$ m, we have

$$B = 0.43\,\frac{T_3 - T_1}{v_2^2}; \quad L = -0.13 + \frac{1}{2B}\left(1 + \sqrt{1 + 2.1B}\right) \qquad \text{m}$$

$$v_* = \frac{0.095\,v_2}{1 + \frac{0.13}{L}} \quad \frac{\text{m}}{\text{sec}}; \qquad q = 0.58\,\frac{v_*(T_1 - T_3)}{1 + \frac{0.65}{L}} \quad \frac{\text{cal}}{\text{cm}^2\text{min}} \tag{51}$$

Examples of computation of the turbulent heat flux q (from data of the Geophysical Institute of the Academy of Sciences expedition of the USSR of 1951) are given in [18]. Computations using specific data show that the scale of L is usually of the order of 10 m, and approaches 3-4 m only in speoific cases with great instability or abrupt inversions. In cases close to isothermy, L reaches values of several tens of meters. The drag velocity v_* is about 8% of the wind velocity at 8 m with unstable stratification, and about 5% with with stable stratification. In the summer in Kazakhstan the turbulent heat flux q reaches 0.25-0.35 cal/cm^2 min on hot sunny days, while it is of the order of 0.06 cal/cm^2 min at night.

Considering that some researchers use the formulae proposed by Budyko [3] and Laikhtman [5] when determining the turbulence characteristics from gradient measurement data, let us derive the relationship between the scale of L and thebasic parameters of theBuyko and Laikhtman formulas. Budyko approximates the wind profiles by the logarithmic law

$$v(z) = \frac{v_*}{\kappa m} \ln \frac{mz}{h_o} \tag{52}$$

where m is a parameter which is a function of atmospheric stratification (with neutral stratification, m reverts to unity). Equating the expressions for $v(z_2)/v(z_1)$, computed from the formulae (43) and (52), we get the ratio

$$\frac{\beta}{L} = \frac{\ln m \quad \ln \frac{z_2}{z_1}}{z_1 \ln \frac{z_2}{h_o} - z_2 \ln \frac{z_1}{h_o} + (z_1 - z_2) \ln m} \tag{53}$$

Taking the limit where $z_2 \to z_1 = H$, we get

$$\beta\,\frac{H}{L} = \frac{\ln m}{1 - \ln m + \ln \frac{h_o}{H}} \tag{54}$$

D.L. Laikhtman approximates the wind profile by the power law

$$v(z) = v(z_1)\frac{z^\delta - h_o^\delta}{z_1^\delta - h_o^\delta} \tag{55}$$

where δ is a parameter which is a function of atmospheric stratification (with neutral stratification, δ reverts to zero). Equating the expression for $v(z_2)/v(z_1)$, computed from formulae (43) and (55), we get the relationship

$$\frac{\beta}{L} = \frac{(z_2^\delta - h_o^\delta)\ln\frac{z_1}{h_o} - (z_1^\delta - h_o^\delta)\ln\frac{z_2}{h_o}}{z_2\,(z_1^\delta - h_o^\delta) - z_1\,(z_2^\delta - h_o^\delta)} \tag{56}$$

Taking the limit as $z_2 \to z_1 = H$ we get

$$\beta\,\frac{H}{L} = \frac{\delta \ln\frac{H}{h_o} - \left[1 - \left(\frac{h_o}{H}\right)^\delta\right]}{1 - \delta - \left(\frac{h_o}{H}\right)^\delta} \tag{57}$$

Taking advantage of the fact that the value of δ is insignificantly small, and expanding the right side of (57) in series according to the δ-exponents, we get the approximation

$$\beta\,\frac{H}{L} \approx -\frac{\delta}{2}\,\frac{\ln^2\frac{h_o}{H}}{1 + \ln\frac{H}{h_o}} \tag{58}$$

4 Asymptotic formulas for the universal function

From formulas (30) and (40) it follows that in a stationary turbulent surface layer, the wind an temperature profiles can be described using one universal function of z/L. Thus, integrating (30) with respect to z and setting $f(\xi) = \int^\xi \frac{\varphi(\xi)\mathrm{d}\xi}{\xi}$ we obtain

$$v(z) = \frac{v_*}{\kappa}\left[f\left(\frac{z}{L}\right) - f\left(\frac{h_o}{L}\right)\right]$$

$$T(z) = T(h_o) + T_*\left[f\left(\frac{z}{L}\right) - f\left(\frac{h_o}{L}\right)\right] \tag{59}$$

In the present section we will investigate the form of the universal function $f(z/L)$ taken as a whole.

Since $\varphi(\xi) \to 1$ when $\xi \to 0$ with small z/L the function $f(z/L)$ is of an asymptotically logarithmic nature

$$f\left(\frac{z}{L}\right) \approx \ln\left|\frac{z}{L}\right| + \text{const.} \qquad \text{when } \left|\frac{z}{L}\right| \ll 1 \tag{60}$$

With large z/L the asymptotic behavior of function $f(z/L)$ will differ in cases of unstable ($L < 0$) or stable ($L > 0$) stratificatlon, since in these cases there are actually two qualitatively different regimes of turbulent motions.

To analyze the case of unstable stratification, first let us examine the limiting case of purely thermal turbulence (with no wind). In this case, due to the lack of an averaged wind, the friction stress, on average will be zero ($v_* = 0$), while the turbulence regime is characterized by only the parameters q and g/T_o (the turbulence receives its energy exclusively from the Instability energy, and therefore is a function only of the degree of instability, characterized by the heat flux $q > 0$ and of the magnitude of the Archimedean forces, characterized by the parameter g/T_o).

We cannot form a length scale from the parameters q and g/T_o; therefore, the regime of purely thermal turbulence is automodular**, i.e. all its characteristics are combinations of q, g/T_o and z. From dimensional considerations we get

$$T(z) = T_\infty + \frac{C}{\kappa^{4/3}} \left(\frac{q}{c_p\,\rho}\right)^{2/3} \left(\frac{gz}{T_o}\right)^{-1/3} \tag{61}$$

where C is the non-dimensional (universal) constant, the factor $\kappa^{-4/3}$ is introduced for convenience, and T_∞ is a constant which has a temperature dimension.

From (61) it is evident that with an increase in height the distribution of temperature approaches isothermy[8] This is natural, since in the case of unstable stratification at great heights, large turbulent elements develop (whose dimensions are limited only by the distance to the earth's surface), bringing about very intense mixing of the air, which leads to an equalization of the temperature profile.

From (61) it follows that the austausch coefficient

$$K = -\frac{q}{c_p\rho\frac{\mathrm{d}T}{\mathrm{d}z}} = \frac{3}{C}\left(\frac{q}{c_p\rho}\right)^{1/3}\left(\frac{g}{T_o}\right)^{1/3}(\kappa z)^{4/3} \tag{62}$$

increases rapidly with height, which is explained by the augmentation of the

**Editorial note: This word is translated by LSG as "self patterning" (with quote marks).

[8]In (61) we are speaking of the approach to "potential isothermy" with an increase in height (see footnote 5).

turbulent elements with an increase in height and simultaneous increase in the intensity of the fluctuations.[9]

Formally, formula (61) can be written

$$\frac{T(z) - T(h_o)}{T_*} = C\left(\frac{z}{L}\right)^{-1/3} - C\left(\frac{h_o}{L}\right)^{-1/3} \tag{63}$$

so that in the case of purely thermal turbulence, the universal function $f(z/L)$ (determined to within an additive constant) has the form $f(z/L) = C\,(z/L)^{-1/3}$ + const.

The case of purely thermal turbulence can be derived from the general case of unstable stratification by passage to the limit with $v_* \to 0$. Here $L \to 0$ and $z/L \to \infty$. Therefore the asymptotic behavior of the universal function $f(z/L)$ is determined by the relationship

$$f(z/L) \sim C\,(z/L)^{-1/3} + \text{const.} \qquad \text{when } z/L \ll -1 \tag{64}$$

This result indicates that at great heights $z \gg L$ (in the surface layer) the turbulent regime, in the case of unstable stratification, is determined mainly by thermal factors (the wind profile is smoothed, and turbulence receives its energy mainly from the energy of turbulent instability, not from the energy of the average motion).

An explanation of the asymptotic behavior of the function $f(z/L)$ when $z \gg L$ in the case of stable stratification, requires that we introduce additional concepts. Turbulence decays in the limiting case of abrupt inversion with a vanishingly weak wind. The existence of large turbulent elements becomes impossible in the case of stable stratification (since they must expend too much energy on opposing the Archimedean forces), and turbulence can exist only in the form of small eddies. Large waves cannot lose stability, which is natural from the point of view of the theory of stability. In this case turbulent exchange between different atmospheric layers is hampered and turbulence takes on a local character; at rather high altitudes $z \gg L$ (or, to put it another way, with strong stability, that is with small $L > 0$) the turbulence characteristics evidently cannot be functions of the distance z to the substrate. This pertains, in particular, to the mixing coefficient K and, accordingly, also to the Richardson number Ri.

Thus, we may consider that in the case of stable stratification with an increase in height z, (or, with an increase in stability, i.e., a decrease in L) the

[9] The concepts here presented on the regime of purely thermal turbulence agree with the system proposed by A. A. Skvortsov [20], with the sole difference that Skvortsov introduces a concept of the discrete spectrum of the scales of turbulent structures, while in the system presented here, the spectrum of the scales is assumed to be continuous.

coefficient of mixing K and the Richardson number Ri tend toward certain constant values. This is natural, since with an increase in stability, K evidently cannot increase, while Ri cannot decrease. Accordingly, there is a (universal) value R of the Richardson number, which is such that when $z/L \gg 1$,

$$Ri \sim R = \text{const.}, \qquad K \sim \kappa v_* L\, R \tag{65}$$

The limiting value of R evidently cannot be greater than the critical value Ri_{cr}, but since, asymptotically, $K \neq 0$, i.e., turbulence does not completely degenerate, R should be less than Ri_{cr}. The limiting value obtained will be called the stationary Richardson number.

From (65) it follows that when $z/L \gg 1$, $f'(\xi) \approx 1/R$, or

$$f(\frac{z}{L}) \approx \frac{1}{R}\,\frac{z}{L} + \text{const.} \tag{66}$$

Then we have

$$v(z) \sim -\frac{1}{R}\,\frac{g}{T_o}\,\frac{q}{c_p\rho}\,\frac{z}{v_*^2} + \text{const.} \tag{67}$$

$$T(z) \sim \frac{1}{R}\,\frac{g}{T_o}\left(\frac{q}{c_p\rho}\right)^2 \frac{z}{v_*^4} + \text{const.} \tag{68}$$

Our formulas (60), (64) and (66) show the behavior of function $f(\xi)$ when $|\xi| \ll 1,\ \xi \ll -1$ and $\xi \gg 1$, respectively.

For an empirical determination of the universal function $f(\xi)$ in a sufficiently broad range of changes in the parameter ξ, using the data of the four expeditions, given in table 2, and determining v_* and L (when $\beta = 0.6$) for each wind profile, we construct the empirical universal function

$$\frac{\kappa}{v_*}\left[v(z) - v\left(\frac{|L|}{2}\right)\right] = f\left(\frac{z}{L}\right) - f(\pm\frac{1}{2})$$

where the plus sign corresponds to stable stratification and the minus sign to unstable stratification.

The empirical points obtained are plotted on the graph in Figure 4. The graph gives convincing evidence of the suitability of the hypotheses of similitude used in the present work; these hypotheses reduce to the existence of a single universal function $f(z/L)$. The empirical points lie along smooth curves with a very small scatter, despite the inaccuracies of the wind measurements and the computation of L and v_* by the approximation methods shown above. Some scatter of the points is noted only in highly stable cases. The drawing shows the limiting behavior of the curve quite well for the case of high stability (approaching a linear profile) and high instability (approaching a constant).

Table 2

Wind profiles determined from data grouped according to stability parameter

Main Geophysical Observatory Expedition 1945; $z_o = 0.2$ cm.

Code No.	No. of Pro-files	100 S	Wind velocity (m/s) at the height (m)						$\frac{v_*}{\kappa}$	$\frac{S}{L}$	L
			0.2	0.5	1.0	2.0	5.0	10.8			
-2	18	-3.47	0.55	0.74	0.94	1.21	1.72	2.40	0.13	0.94	0.68
-1	6	-0.52	1.21	1.49	1.76	2.10	2.78	3.80	0.25	0.57	1.0
0	30	0.25	2.41	2.90	3.25	3.60	4.10	4.56	0.50	0.01	46.2
1	21	0.56	1.69	2.00	2.24	2.43	2.68	2.85	0.36	-0.07	-8.2
2	10	1.49	1.32	1.58	1.70	1.82	1.97	2.02	0.27	-0.11	-5.6

Main Geophysical Observatory Expedition 1947; $z_o = 0.5$ cm.

Code No.	No. of Pro-files	100 S	Wind velocity (m/s) at the height (m)						$\frac{v_*}{\kappa}$	$\frac{S}{L}$	L
			0.5	1.0	2.0	5.0	9.0	14.5			
-4	8	-5.73	0.74	0.91	1.08	1.40	1.61	1.86	0.16	0.23	2.6
-3	13	-0.91	1.09	1.32	1.48	1.76	1.88	2.08	0.25	0.04	16.2
-2	9	-.37	1.24	1.50	1.67	1.93	2.03	2.03	0.28	0.002	300.0
-1	13	-0.18	1.68	2.00	2.22	2.57	2.73	2.92	0.37	0.01	-54.5
0	22	0	1.90	2.24	2.51	2.84	2.95	3.26	0.42	-0.02	-30.0
1	37	0.09	3.36	3.93	4.41	4.88	5.15	5.57	0.75	-0.65	-11.1
2	41	0.26	2.66	3.15	3.50	3.80	3.98	4.18	0.60	-0.07	-9.1
3	38	0.44	2.44	2.91	3.18	3.54	3.63	3.84	0.56	-0.07	-8.8
4	19	0.57	2.24	2.61	2.85	3.12	3.16	3.36	0.50	-0.08	-7.0
5	24	0.74	2.02	2.35	2.60	2.81	2.82	3.00	0.45	-0.08	-6.4
6	14	0.55	1.85	2.13	2.34	2.56	2.60	2.75	0.41	-0.09	-6.8
7	9	1.47	1.32	1.53	1.64	1.83	1.80	1.89	0.29	-0.11	-5.7

Main Geophysical Observatory Expedition 1950; $z_o = 0.8$ cm.

Code No.	No. of Pro-files	100 S	Wind velocity (m/s) at the height (m)						$\frac{v_*}{\kappa}$	$\frac{S}{L}$	L
			0.5	1.0	2.0	4.0	8.0	15.0			
-6	19	-8.42	0.54	0.65	0.80	1.00	1.31	1.80	0.12	0.46	1.3
-5	9	-1.92	0.89	1.04	1.22	1.50	1.94	2.50	0.20	0.29	2.1
-4	16	-1.18	1.05	1.25	1.45	1.76	2.16	2.58	0.25	0.18	3.3
-3	15	-0.46	1.52	1.79	2.04	2.34	2.74	3.25	0.36	0.09	6.4
-2	17	-0.24	1.80	2.12	2.42	2.76	3.19	3.66	0.43	0.06	10.3
-1	14	-0.13	2.02	2.38	2.70	3.06	3.50	4.00	0.49	0.04	14.3
0	25	-0.03	2.76	3.21	3.69	4.14	4.59	5.00	0.66	-0.01	-66.7
1	27	-0.09	3.35	3.90	4.48	5.00	5.52	6.08	0.75	-0.01	-66.7
2	25	0.14	2.48	2.90	3.33	3.94	4.10	4.40	0.60	-0.03	-20.0
3	29	0.22	2.40	2.80	3.20	3.55	3.88	4.15	0.58	-0.04	-17.1
4	26	0.26	2.46	2.86	3.25	3.60	3.88	4.10	0.60	-0.05	-11.5
5	26	0.29	2.40	2.75	3.10	3.45	3.80	4.10	0.57	-0.03	-12.4
6	32	0.36	2.28	2.50	2.84	3.16	3.46	3.42	0.52	-0.04	-15.0
7	116	0.46	2.04	2.38	2.69	2.98	3.20	3.40	0.50	-0.05	-11.5
8	18	0.66	1.68	1.94	2.19	2.40	2.62	2.80	0.41	-0.05	-12.5
9	22	0.25	1.44	1.65	1.88	2.04	2.20	2.34	0.35	-0.06	-10.2
10	15	1.22	1.25	1.44	1.61	1.76	1.90	2.00	0.30	-0.06	-9.2
11	19	1.83	1.00	1.14	1.30	1.43	1.53	1.56	1.56	-0.08	-7.7
12	15	4.06	0.73	0.84	0.94	1.02	1.08	1.10	1.18	-0.09	-6.4

Geophysical Institute of the Academy of Sciences Expedition 1951; $z_o = 1.0$ cm.

Code No.	No. of Pro-files	100 S	Wind velocity (m/s) at the height (m)						$\frac{v_*}{\kappa}$	$\frac{S}{L}$	L
			0.5	1.0	2.0	4.0	8.0	15.0			
-2	6	-.64	1.32	1.65	1.91	2.31	2.93	3.91	0.33	0.32	1.8
-1	11	-.21	1.83	2.14	2.55	3.05	3.71	4.62	0.45	0.20	3.0
0	10	0.02	2.97	3.53	4.04	4.72	5.20	5.86	0.76	0.02	24.0
1	7	0.17	4.01	4.64	5.25	5.80	6.41	6.88	1.01	-0.03	-18.2
2	19	0.36	3.09	3.60	4.04	4.45	4.77	5.07	0.79	-0.06	-10.3
3	8	0.81	2.23	2.55	2.86	3.10	3.28	3.45	0.56	-0.08	-7.6

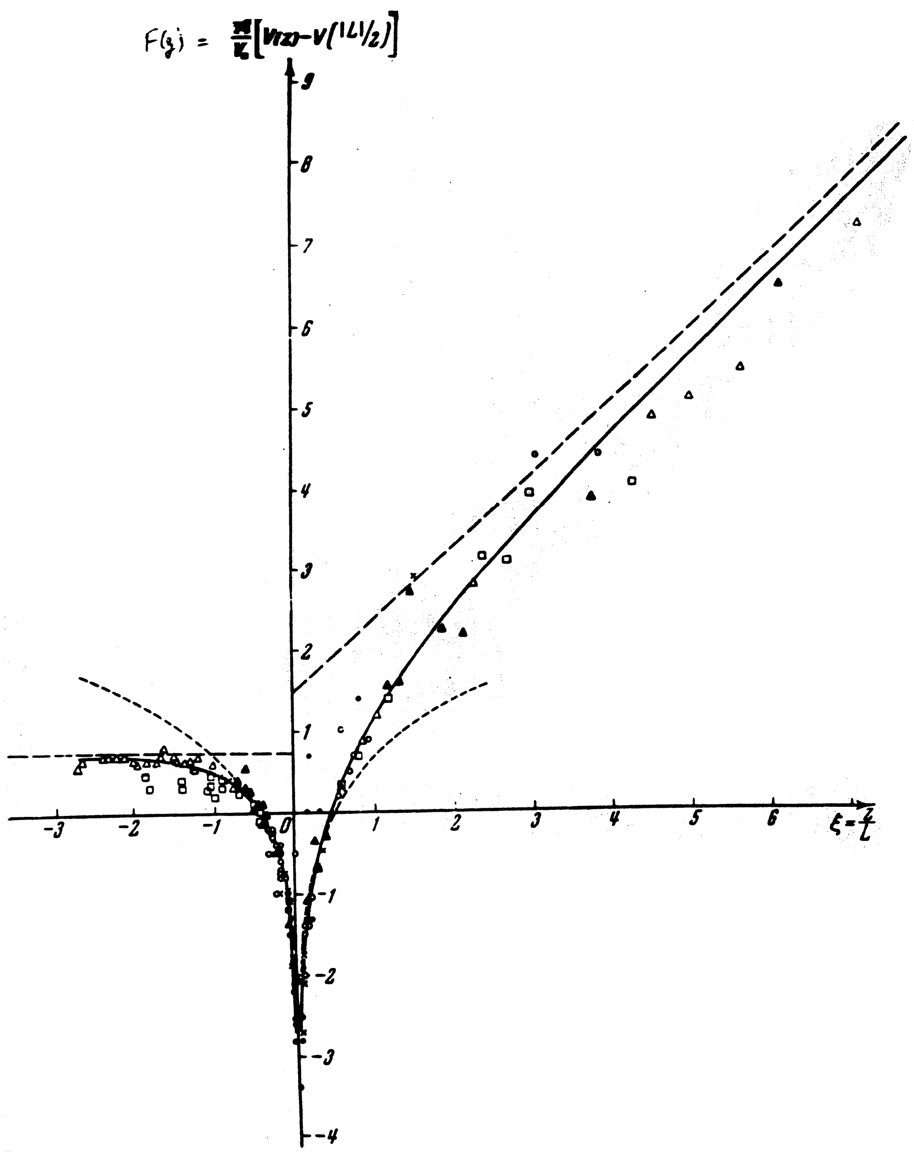

Figure 4: Distribution of wind velocity in non-dimensional coordinates

References

[1] Sapozhnikova, S. A.: Wind velocity variation with height in the lower layer of the atmosphere. Transactions, N.I.U. G.U.G.M.S. No. 33, 1946)

[2] Laikhtman, D. L. and A. F. Chudinovskii: Physic of the surface layer of the atmosphere. (G.T.T.I., 1949, 254 pp.)

[3] Budyko, M.I.: Evaporation in natural conditions. (Gidrometeorizdat, 1948, 135 pp.)

[4] Timofeev, M. T.: On the methods of determination of the components of the heat balance of the underlying surface. (Transactions G.G.O. Issue 27(89) 1951)

[5] Laikhtman, D. L.: Wind profile and the exchange in the surface layer of the atmosphere (Bull. A. Sc. USSR, Geogr. & Geophys. Series 8(1), 1944)

[6] Sutton, 0. G.: Atmospheric Turbulence. (London, 1949. 107 pp.)

[7] Obukhov, A.M.: Turbulence in the temperature-heterogeneous atmosphere (Trans. Inst. Theoretical Geophysics, 1, 95-115. 1948)

[8] Monin, A.S.: Temperature regime in the surface layer of air. (Information Bull. G.U.G.M.S. No. 1, 13-27, 1950.)

[9] Sheppard, P.A.: The aerodynamic drag of the earth's surface and the value of von Karman's constant in the lower atmosphere, (Proc. Roy. Soc. A, 188: 201-222, 1947)

[10] Paeschke, W.: Experimental investigation on the problems of roughness and stability in the surface layer of the atmosphere. (Beit. Phys. fr Atm. 24(3), 1937)

[11] Konstantinov, A. R. Computation of evaporation from agrictiltural fields, taking into account the effects of forest strips (Trans. State Hydrological Institute, 34(88), 1952

[12] Obukhov, A.M.: Characteristics of the microstructure of the wind in the surface layer of the atmosphere, (Bull. A. Sc. USSR Geophys Series 3, 49-68,1951),

[13] Krechmer, S.L.: . Investigations of the microfluctuations of the temperature field in the atmosphere, (D.A.N. USSR, 84(1), 55-58, 1952)

[14] Landau, L.D. and Lifshitz, E.W.: Mechnics of continuous media. (Chapter 5). O.T.T.L. 1944.

[15] Sapozhnikova, S. A.: Some characteristics of the distribution of temperature, humidity and wind in the surface layer of the atmosphere. (Trans. N.I.U.,G.U.G.M.S. USSR, Ser. 1 Meteorology, No. 39, 45-57, 1947.

[16] Sapozhnikova, S. A.: Methods of investigating turbulent exchange by means of observational data in the lower atmosphere. (Trans. G.G.O. 16(78): 25-51,1949).

[17] Sapozhnikova, S. A.: Investigation of the effect of forest plantings on the wind velocity, radiation balance and turbulent exchange in a field. (Trans. G.G.O. 19(91),1952)

[18] Monin, A.S.: 0n the mechanism of heating of the air in open steppe. (Climatological and microclimatological investigations in pre-Caspian lowland. A. Sc. USSR, 1953, pp. 100-123)

[19] Halstead, M.: A stability term in the wind gradient equation. (Trans. Am. Geoph. Un. 1943)

[20] Skvortsov, A.A.: Evaporation and exchange in the surface layer of the atmosphere. (Trans. Inst. of Energetics, A.Sc. U.Z.S.S.R. Tashkent, 1947)